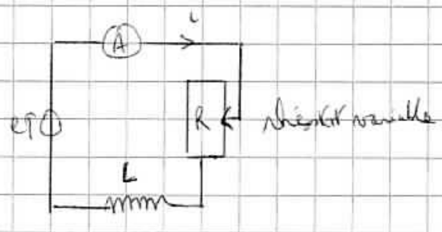
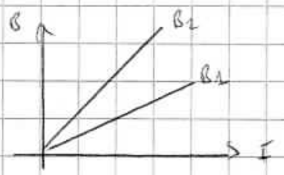
Champ magnétique

I) 1)



2) a)



b) $B = KI$ avec $K_1 = 637\ \mu T.A^{-1}$

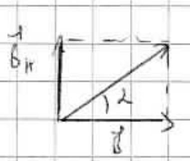
$K_2 = 1260\ \mu T.A^{-1}$

c) $\frac{K_2}{K_1} = 1{,}98 \simeq \frac{n_2}{n_1} = 2$

$\Rightarrow \boxed{B = \mu n I}$

3) $\frac{|\mu - \mu_0|}{\mu_0} = 4{,}2\cdot10^{-2} = \underline{4{,}2\ \%}$

II) 1)



$\tan\alpha = \frac{B_H}{B}$ avec $B = \mu_0 \frac{N_1}{L_1} I = 1{,}13.10^{-5}\ T$

$\Rightarrow \underline{\alpha = 60{,}5°}$

2) $\frac{N_2}{L_2} > \frac{N_1}{L_1} \Rightarrow B_2 > B_1$ donc $B = B_2 \pm B_1$

$\tan\alpha = \frac{B_H}{B_2 \pm B_1} \Rightarrow \boxed{I = \frac{B_H}{\mu_0 \left(\frac{N_2}{L_2} \pm \frac{N_1}{L_1}\right) \tan\alpha}}$ $\underline{I = 31{,}8\ mA}$ ou $\underline{I = 53\ mA}$

Étude de quelques pendules

I) 1)
$\vec{OM} = l\vec{u_r} \qquad \vec{v} = l\dot{\theta}\vec{u_\theta}$

$\vec{\sigma} = \vec{OM} \wedge \vec{p} = \vec{OM} \wedge m\vec{v} = l\vec{u_r} \wedge ml\dot{\theta}\vec{u_\theta} = ml^2\dot{\theta}\,\vec{u_z}$

$\frac{d\vec{\sigma}}{dt} = ml^2\ddot{\theta}\,\vec{u_z}$

$\vec{M_o} = \vec{OM} \wedge \vec{P} + \vec{OM} \wedge \vec{T} = \vec{OM} \wedge \vec{P} = l\vec{u_r} \wedge mg\vec{u_x}$

$= mgl\sin\theta\,(-\vec{u_z})$

$\frac{d\vec{\sigma}}{dt} = \vec{M_o} \Rightarrow ml^2\ddot{\theta} = -mgl\sin\theta \Rightarrow \boxed{\ddot{\theta} + \frac{g}{l}\sin\theta = 0}$

2) petites oscillations $\Rightarrow \ddot{\theta} + \frac{g}{l}\theta = 0 \qquad \omega_0 = \sqrt{\frac{g}{l}}$

$\Rightarrow \theta = A\cos(\omega_0 t) + B\sin(\omega_0 t)$

à $t=0 \quad \theta = \theta_0 = A$

$v = v_0 = l\dot{\theta}_0 \Rightarrow \dot{\theta}_0 = \frac{v_0}{l} = B\omega_0 \Rightarrow B = \frac{v_0}{l\omega_0}$

$\Rightarrow \boxed{\theta = \theta_0\cos(\omega_0 t) + \frac{v_0}{l\omega_0}\sin(\omega_0 t)}$ avec $\omega_0 = \sqrt{\frac{g}{l}}$

II) 1)
$\boxed{\begin{array}{l} r = l\sin\alpha \\ z = -l\cos\alpha \end{array}} \qquad (z<0)$

2) $\vec{OM} = r\vec{e_r} + z\vec{e_z}$ avec $z<0$

$\vec{v} = r\dot{\theta}\vec{e_\theta}$

$\vec{\sigma} = \vec{OM} \wedge m\vec{v} = m(r\vec{e_r} + z\vec{e_z}) \wedge r\dot{\theta}\vec{e_\theta} = mr^2\dot{\theta}\vec{e_z} - mrz\dot{\theta}\vec{e_r}$

$\frac{d\vec{\sigma}}{dt} = mr^2\ddot{\theta}\vec{e_z} - mrz\ddot{\theta}\vec{e_r} - mrz\dot{\theta}^2\vec{e_\theta}$

$\vec{M_o} = \vec{OM} \wedge \vec{P} + \vec{OM} \wedge \vec{T} = \vec{OM} \wedge \vec{P} = (r\vec{e_r} + z\vec{e_z}) \wedge -mg\vec{e_z} = +mgr\vec{e_\theta}$

$\frac{d\vec{\sigma}}{dt} = \vec{M_o} \Rightarrow \begin{cases} -mrz\ddot{\theta} = 0 \\ -\cancel{m}\cancel{r}z\dot{\theta}^2 = \cancel{m}g\cancel{r} \\ mr^2\ddot{\theta} = 0 \end{cases} \Rightarrow \underline{\ddot{\theta} = 0} \Rightarrow$ mvt uniforme

3) $\dot{\theta}^2 = \frac{g}{-z} \Rightarrow \omega = \sqrt{\frac{-g}{z}} = \sqrt{\frac{g}{l\cos\alpha}} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{l\cos\alpha}{g}}} \simeq 2\pi\sqrt{\frac{l}{g}}$ si α faible

Obtention NaCl dans eau de mer

1) a) L'eau de mer contient d'autres ions ($MgCl_2$ par exemple)

b) vent : emporte la vapeur d'eau

soleil : provoque l'évaporation par apport d'énergie

2) $c_m = 27{,}2 \text{ g L}^{-1}$ $\quad M(NaCl) = 58{,}5 \text{ g mol}^{-1}$

$\Rightarrow \underline{C = 0{,}46 \text{ mol L}^{-1}}$

3) a) Conservation de la masse en NaCl (avant précipitation des premiers grains)

$$m_{NaCl} = c\, V_{initial} = c_{sat}\, V_{limite}$$

avec $K_S = [Na^+][Cl^-] = c_{sat}^2 \Rightarrow c_{sat} = \sqrt{K_S} = \underline{6 \text{ mol L}^{-1}}$ (351 g L^{-1})

b) d'où $\dfrac{V_{initial} - V_{limite}}{V_{initial}} = 1 - \dfrac{c}{c_{sat}} = 1 - \dfrac{0{,}46}{6} = \underline{92{,}3\%}$

c) $m_{final} = c_{sat} V_{final} = 0{,}2\, m_{initial} = 0{,}2\, c\, V_{initial}$

$$\Rightarrow \frac{V_{final}}{V_{initial}} = 0{,}2 \frac{c}{c_{sat}}$$

$$\Rightarrow \frac{V_{initial} - V_{final}}{V_{initial}} = 1 - 0{,}2 \frac{c}{c_{sat}} = \underline{98{,}5\%}$$

d) effet d'ion commun : $[Cl^-]$ augmente $\Rightarrow$ K_s atteint plus facilement.

e) $[Cl^-]_i = \dfrac{27{,}2}{58{,}5} + 2\dfrac{3{,}8}{95{,}3} = 0{,}54 \text{ mol L}^{-1}$

$[Na^+]_i = 0{,}46 \text{ mol L}^{-1}$

$$K_s = [Cl^-][Na^+] = \frac{[Cl^-]_i \times V_{initial}}{V_{limite}} \times \frac{[Na^+]_i \times V_{initial}}{V_{limite}}$$

$$\Rightarrow \frac{V_{initial}}{V_{limite}} = \sqrt{\frac{36}{0{,}54 \times 0{,}46}} = 12 \Rightarrow \frac{V_{initial} - V_{limite}}{V_{initial}} = 1 - \frac{1}{12} = \underline{91{,}7\%}$$

Second principe

1) $p_0 V_A = RT_0 \Rightarrow V_A = \frac{RT_0}{p_0} = \underline{24,9\ L}$

$p_1 V_{B_1} = RT_0 \Rightarrow V_{B_1} = \frac{RT_0}{p_1} = \underline{8,3\ L}$

2) $p_1 V_{B_1}^{\gamma} = p_0 V_{C_1}^{\gamma} \Rightarrow V_{C_1} = V_{B_1}\left(\frac{p_1}{p_0}\right)^{\frac{1}{\gamma}} = \underline{18,2\ L}$

$p_1^{1-\gamma} T_0^{\gamma} = p_0^{1-\gamma} T_{C_1}^{\gamma} \Rightarrow T_{C_1} = T_0\left(\frac{p_1}{p_0}\right)^{\frac{1-\gamma}{\gamma}} = \underline{219,2\ K}$

3) Isotherme AB_1 $\quad \Delta U = 0 \Rightarrow W_{AB_1} = -Q_{AB_1} = -\int p\,dV = -nRT_0 \ln\left(\frac{V_{B_1}}{V_A}\right) = 2742\ J$

Adiabatique B_1C_1 $\quad Q_{B_1C_1} = 0 \quad W_{B_1C_1} = \Delta U = n\frac{R}{\gamma-1}(T_{C_1} - T_{B_1}) = -1680\ J$

$\Rightarrow \underline{Q_{AC_1} = -2742\ J}$

$\underline{W_{AC_1} = 1062\ J}$

4) Réversible $\Rightarrow \underline{S^c_{AC_1} = 0}$

thermostat entre A et B_1 $\Rightarrow S^e_{AC_1} = \frac{-Q_{th}}{T_0} = +\frac{Q_{AB_1}}{T_0} \Rightarrow \underline{S^e_{AC_1} = -9,1\ JK^{-1}} \quad (S^e_{AC_1} = -\Delta S_{th})$

$\Delta S_{AC_1} = S^e_{AC_1} + S^c_{AC_1} = \underline{-9,1\ JK^{-1}}$

5) $T_{C_4} = T_{C_3}\left(\frac{p_1}{p_0}\right)^{\frac{1-\gamma}{\gamma}} = T_{C_2}\left(\frac{p_1}{p_0}\right)^{\frac{2-2\gamma}{\gamma}} \Rightarrow \underline{T_{C_4} = 3^{\frac{2-2\gamma}{\gamma}} T_{C_2} = 0,53\,T_{C_2}}$

$\frac{T_{C_4}}{T_{C_2}} = \frac{V_{C_4}}{V_{C_2}} \Rightarrow \underline{V_{C_4} = 0,53\,V_{C_2}} = V_{C_1}\left(\frac{p_1}{p_0}\right)^{\frac{2-2\gamma}{\gamma}} \quad (V_{C_2} = 1,87\,V_{C_4})$

6) $V \downarrow$ et $T \downarrow$ $\Rightarrow$ le gaz est comprimé et refroidi

7) $V'_{B_1} = V_{B_1} = \underline{8,3\ L}$

$\Delta U_{B_1C_1} = 0 + W_{B_1C_1} \Rightarrow n\frac{R}{\gamma-1}(T'_{C_1} - T_0) = -p_0(V'_{C_1} - V_{B_1}) = -nRT'_{C_1} + p_0\frac{nRT_0}{p_1}$

$T'_{C_1}\frac{\gamma}{\gamma-1} = \frac{T_0}{\gamma-1} + \frac{p_0}{p_1}T_0 \Rightarrow T'_{C_1} = \frac{T_0}{\gamma} + \frac{\gamma-1}{\gamma}\frac{p_0}{p_1}T_0 = \frac{T_0}{\gamma}\left(1 + (\gamma-1)\frac{p_0}{p_1}\right)$

$\Rightarrow \underline{T'_{C_1} = 243\ K}$

8) Monotherme $\quad \Delta U = 0 \Rightarrow W'_{AB_1} = -Q_{AB_1} = -p_1(V_{B_1} - V_A) = 4980\ J$

Adiab $\quad Q'_{B_1C_1} = 0 \quad W'_{B_1C_1} = \Delta U = n\frac{R}{\gamma-1}(T'_{C_1} - T'_{B_1}) = -1185\ J$

$\Rightarrow \underline{Q'_{AC_1} = -4980\ J}$

$\underline{W'_{AC_1} = 3794\ J}$

9) $S'^e_{AC_1} = -\Delta S_{thermostat} = \frac{-Q_{th}}{T_0} = \frac{Q_{AB_1}}{T_0} = \underline{-16,6\ JK^{-1}}$

$\Delta S'_{AC_1} = \frac{\gamma R}{\gamma-1}\ln\left(\frac{T_{C_1}}{T_0}\right) = \underline{-6,1\ JK^{-1}}$

$\Rightarrow S'^c_{AC_1} = \Delta S'_{AC_1} - S^e_{AC_1} = \underline{10,5\ JK^{-1}}$

Réacteur fermé avec double enveloppe

1) $P_{th}\,dt$ énergie reçue par chauffage

$\rho V C_p\,dT$ énergie emmagasinée par le système

$-UA(T-T_p)\,dt$ énergie algébriquement reçue de la part de l'extérieur (pertes si $T > T_p$)

$$\boxed{\rho V C_p\,dT = P_{th}\,dt - UA(T-T_p)\,dt}$$

2) régime permanent $\frac{dT}{dt} = 0 \Rightarrow P_{th} = UA(T-T_p) \Rightarrow \boxed{T-T_p = \frac{P_{th}}{UA}}$

3) $(T-T_p)_{eq} = 40\,K$ $\qquad U = \frac{P_{th}}{A(T-T_p)} = \underline{300\ W\,m^{-2}K^{-1}}$

4) $\frac{dT}{dt} = \frac{P_{th}}{\rho V C_p} + \frac{UA}{\rho V C_p}(T_p - T) \Rightarrow \boxed{\begin{array}{l} s = \frac{P_{th}}{\rho V C_p} \\ \tau_c = \frac{\rho V C_p}{UA} \end{array}}$ $\qquad \tau_c$ homogène à un temps

5) $\frac{dT}{dt} = \frac{UA}{\rho V C_p}(T_p - T) = +\frac{T_p - T}{\tau_c}$

6) $\frac{dT}{dt} + \frac{T}{\tau_c} = \frac{T_p}{\tau_c} \Rightarrow T = A\exp\left(-\frac{t}{\tau_c}\right) + T_p$

à $t=0$ $\quad T = T_{max}$

$\Rightarrow A = T_{max} - T_p$

$$\Rightarrow \boxed{T - T_p = (T_{max} - T_p)\exp\left(-\frac{t}{\tau_c}\right)}$$

7) $\ln(T-T_p) = \ln(T_{max} - T_p) - \frac{t}{\tau_c}$

$\Rightarrow \frac{1}{\tau_c} = 1{,}33\cdot 10^{-2} \Rightarrow \underline{\tau_c = 75{,}2\ s} = \frac{\rho V C_p}{UA}$

$\Rightarrow U = \frac{\rho V C_p}{A\tau_c} = \underline{299{,}25\ W\,m^{-2}K^{-1}}$ $\qquad$ ok