Frenquences audibles / Mécaniques (1)

I] 1°] $s_1 = S_{1m} \sin(2\pi f_1 t)$

$s_2 = S_{2m} \sin(2\pi f_2 t)$

$s = k s_1 s_2 = \frac{k S_{1m} S_{2m}}{2}\left[\cos(2\pi(f_1 - f_2)t) - \cos(2\pi(f_1+f_2)t)\right]$

$\Rightarrow$ 2 fréquences $\boxed{f_1 - f_2 = 440 \text{ Hz}}$ audible

$\boxed{f_2 + f_1 = 160\,440 \text{ Hz}}$ inaudible

I] 2°] Filtre passe bas

I] 3°] a] $u_{C_0} = L_0 \frac{di}{dt}$ avec $i = -C_0 \frac{du_{C_0}}{dt}$ $\rightarrow$ $\boxed{\frac{du_{C_0}}{dt} + \frac{1}{L_0 C_0} u_{C_0} = 0}$

I] 3°] b] $\omega_0 = \frac{1}{\sqrt{L_0 C_0}}$ $\Rightarrow$ $\boxed{f_2 = \frac{1}{2\pi\sqrt{L_0 C_0}}}$

I] 4] a] Condensateurs en parallèle $i = i_1 + i_2 = C_0 \frac{du}{dt} + C_{h_1}\frac{du}{dt} = (C_0 + C_{h_1})\frac{du}{dt}$

$\Rightarrow C_{éq} = C_0 + C_{h_1}$

$\Rightarrow \boxed{f_1 = \frac{1}{2\pi\sqrt{L_0(C_0 + C_{h_1})}}}$

I] 4] b] $\boxed{f_1 \pm f_2 = \frac{1}{2\pi}\left(\frac{1}{\sqrt{L_0 C_0}} \pm \frac{1}{\sqrt{L_0(C_0 + C_{h_1})}}\right)}$ $\underline{20 \text{ kHz} \leq f_c \leq f_1 + f_2}$

I] 5°] a] $T(jf) = \frac{\frac{1}{jC\omega}}{R + \frac{1}{jC\omega}} = \boxed{\frac{1}{1 + jRC\omega}}$

$\left.\begin{array}{l}\omega \to 0 \quad T(jf) \to 1 \\ \omega \to \infty \quad T(jf) \to 0\end{array}\right\} \Rightarrow$ filtre passe bas

$|T| = \frac{1}{\sqrt{1 + R^2C^2\omega^2}}$ $\Rightarrow |T|_{max} = 1$

$|T(jf_c)| = \frac{1}{\sqrt{1 + R^2C^2\omega_c^2}} = \frac{1}{\sqrt{2}} \Rightarrow R^2C^2\omega_c^2 = 1 \Rightarrow \omega_c = \frac{1}{RC}$

$\Rightarrow \boxed{f_c = \frac{1}{2\pi RC}}$

I] 5] b] $f_c = 20 \text{ kHz} \Rightarrow R = \frac{1}{2\pi f C} = \underline{796\,\Omega}$ (max)

I] 6] $x = a \log f + b$

x	44	32	20	8
$\log f$	2,04	2,34	2,64	2,94

$x = 125{,}73 - 40 \log f$ avec x en cm

I] 2] x augmente f diminue ⇒ plus grave. Il faut éloigner la main

$40 \to 1760$ $\quad 1760 = 40 \times 2^x \Rightarrow x = \dfrac{\log 1760 - \log 40}{\log 2} = 5{,}45$

$\Rightarrow$ 5,5 octaves

$f \to 2f \quad x' = 125{,}73 - 40 \log(2f) \Rightarrow \Delta x = x' - x = 40 \log 2 = 12$ cm

II] 1) par analogie $\boxed{f = \dfrac{1}{2\pi\sqrt{L_1(C_1 + C_2)}}}$

II] 1b) $u = \dfrac{i}{jC\omega}$ et $i = jC\omega u$ $\quad \omega \to 0 \quad i \to 0$ –||– ⇔ –/–

$\omega \to \infty \quad u \to 0$ –||– ⇔ ——

	$\omega \to 0$	$\omega \to \infty$	Nature
Montage 1 :	$u_s = 0$	$u_s = 0$	passe bande
Montage 2 :	$u = -Ri$	$u_s = 0$	passe bas
Montage 3 :	$u_s = 0$	$u_s = -Ri$	passe haut
Montage 4 :	$U_s = 0$	$u_s = 0$	passe bande

II] 3] a) H_0 gain max (pour $f = f_0$)

Q facteur de qualité

f_0 fréquence de résonance :

II] 3] b) $|H(jf_c)| = \dfrac{H_{max}}{\sqrt{2}} \Rightarrow \dfrac{H_0}{\sqrt{1 + Q^2\left(\frac{f_c'}{f_0} - \frac{f_0}{f_c'}\right)^2}} = \dfrac{H_0}{\sqrt{2}} \Rightarrow Q^2\left(\dfrac{f_c'}{f_0} - \dfrac{f_0}{f_c'}\right)^2 = 1$

$\Rightarrow \dfrac{f_c'}{f_0} - \dfrac{f_0}{f_c'} = \pm \dfrac{1}{Q} \Rightarrow f_c'^2 \pm \dfrac{1}{Q} f_c' f_0 - f_0^2 = 0$

$\Delta = \dfrac{f_0^2}{Q^2} + 4f_0^2$

$\Rightarrow f_c' = \dfrac{\pm \frac{f_0}{Q} + \sqrt{\frac{f_0^2}{Q^2} + 4f_0^2}}{2}$

$\boxed{\Delta f' = \dfrac{f_0}{Q}}$

II] 4] $\underline{H} = \dfrac{\underline{u_s}}{\underline{u_e}} = \dfrac{\frac{1}{\underline{Y}}\underline{i}}{-\underline{Z}\,\underline{i}} = \dfrac{1}{-\underline{Z}\,\underline{Y}}$ avec $\underline{Z} = R + \dfrac{1}{jC\omega}$

$\underline{Y} = \dfrac{1}{R} + jC\omega$

$$\underline{H} = \frac{1}{-\left(R + \frac{1}{jC\omega}\right)\left(\frac{1}{R} + jC\omega\right)} \Rightarrow \underline{H} = \frac{-1}{2 + j\left(RC\omega - \frac{1}{RC\omega}\right)}$$

$$\Rightarrow \underline{H} = \frac{-1/2}{1 + j\frac{1}{2}\left(RC\omega - \frac{1}{RC\omega}\right)}$$

$$\boxed{\begin{aligned} H_0 &= -\frac{1}{2} \\ Q &= \frac{1}{2} \\ f_0 &= \frac{1}{2\pi RC} \end{aligned}}$$

II 3 5) $f'_{c\,min} = \dfrac{1}{2\pi\sqrt{L_1 C_M}}$ $\quad f'_{c\,max} = \dfrac{1}{2\pi\sqrt{L_1 C_m}}$

$$f_0 = \frac{f'_{c\,min} + f'_{c\,max}}{2} \qquad Q = \frac{f_0}{\Delta f'} = \frac{\frac{1}{2\pi}\left(\frac{1}{\sqrt{L_1 C_M}} + \frac{1}{\sqrt{L_1 C_m}}\right)}{2\left(\frac{\frac{1}{\sqrt{L_1 C_m}} - \frac{1}{\sqrt{L_1 C_M}}}{2\pi}\right)}$$

$$\Rightarrow \boxed{Q = \frac{\sqrt{C_m} + \sqrt{C_M}}{2\left(\sqrt{C_M} - \sqrt{C_m}\right)}} \qquad Q = \frac{19}{2 \times 1} = \underline{9{,}5}$$

Alcalinité d'une eau

I) 1) a) $CO_3^{2-} + H_2O \rightleftarrows HCO_3^- + OH^-$ $\quad K = 10^{-3,7} = \frac{Ke}{Ka}$

b)

10^{-3}	–	0	0
$10^{-3}-x$		x	x

$\frac{x^2}{10^{-3}-x} = 10^{-3,7} \Rightarrow x = 3,6 \cdot 10^{-4}$ mol L^{-1}

$[CO_3^{2-}] = 6,4 \cdot 10^{-4}$ mol L^{-1}

$[HCO_3^-] = 3,6 \cdot 10^{-4}$ mol L^{-1} $\quad pH = pKa + \log\frac{[CO_3^{2-}]}{[HCO_3^-]} = 10,6$

2) a) $CO_3^{2-} + H_2O \rightleftarrows HCO_3^- + OH^-$ $\quad K = 10^{-3,7}$

10^{-3}		0	10^{-4}
$10^{-3}-x$		x	$10^{-4}+x$

$\frac{x(10^{-4}+x)}{10^{-3}-x} = 10^{-3,7}$

$\Rightarrow 10^{-4}x + x^2 = 10^{-6,7} - 10^{-3,7}x$

$\Rightarrow x^2 + 2,99 \cdot 10^{-4}x - 1,99 \cdot 10^{-7} = 0 \Rightarrow x = 3,2 \cdot 10^{-4}$ mol L^{-1}

$\Rightarrow [OH^-] = 4,2 \cdot 10^{-4}$ mol L^{-1} $\Rightarrow [H_3O^+] = 2,38 \cdot 10^{-11} \Rightarrow pH = 10,6$

b) Pas de variation perceptible car l'apport de OH^- est modéré.

II) 1) H_3O^+ : acide

HSO_4^- : acide $\quad$ (minoritairement SO_4^{2-} base)

2) a) $OH^- + H_3O^+ \rightarrow 2H_2O$ $\quad K = 10^{14}$

$OH^- + HSO_4^- \rightarrow SO_4^{2-} + H_2O$ $\quad K = 10^{12}$

$CO_3^{2-} + H_3O^+ \rightarrow HCO_3^- + H_2O$ $\quad K = 10^{10,3}$

$CO_3^{2-} + HSO_4^- \rightarrow SO_4^{2-} + HCO_3^-$ $\quad K = 10^{8,3}$

b) $HCO_3^- + H_3O^+ \rightarrow H_2CO_3 + H_2O$ $\quad K = 10^{6,4}$

$HCO_3^- + HSO_4^- \rightarrow H_2CO_3 + SO_4^{2-}$ $\quad K = 10^{4,4}$

c) $K > 10^3$ donc quasi totales.

3) V_A correspond à $x_1 + y_1$
V_B correspond à y_1 $\quad \Rightarrow \quad V_A > V_B$

4)
$$\begin{cases} C_2V_A + C_1V_A = [OH^-]_0 V_0 + [CO_3^{2-}]_0 V_0 \\ \underbrace{C_2V_B}_{\text{en } H_3O^+} + \underbrace{C_1V_B}_{\text{et en } HSO_4^-} = [CO_3^{2-}]_0 V_0 \end{cases}$$

$$\Rightarrow [CO_3^{2-}]_0 = \frac{2C_1V_B}{V_0} = \underline{9{,}6 \cdot 10^{-4}\ mol\ L^{-1}}$$

$$\Rightarrow [OH^-]_0 = \frac{2C_1(V_A - V_B)}{V_0} = \underline{4 \cdot 10^{-3}\ mol\ L^{-1}}$$

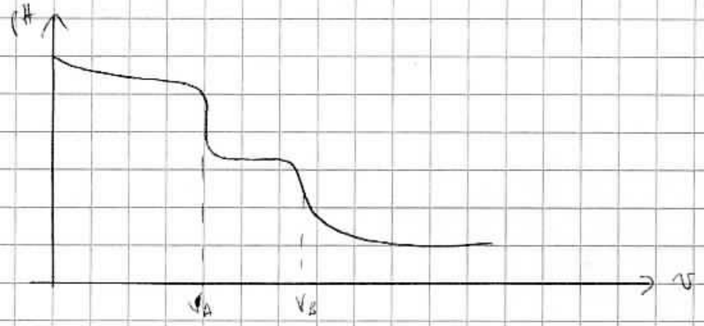
5)
$$10^{-3,7} = \frac{x(4 \cdot 10^{-3} + x)}{9{,}6 \cdot 10^{-4} - x} \Rightarrow x^2 + 4 \cdot 10^{-3} x = 1{,}91 \cdot 10^{-7} - 1{,}99 \cdot 10^{-4} x$$

$$\Rightarrow x^2 + 4{,}2 \cdot 10^{-3} x - 1{,}91 \cdot 10^{-7} = 0$$

$$\Rightarrow x = 4{,}5 \cdot 10^{-5}\ mol\ L^{-1}$$

$$\Rightarrow [OH^-] = 4{,}045 \cdot 10^{-3}\ mol\ L^{-1} \Rightarrow [H_3O^+] = 2{,}47 \cdot 10^{-12} \Rightarrow \underline{pH = 11{,}6}$$

6)



Vibrations d'une molécule

1) $E_p(x) = D\left(\exp(-2\alpha(x-\beta)) - 2\exp(-\alpha(x-\beta))\right)$

$\dfrac{dE_p}{dx} = D\left\{-2\alpha\, e^{-2\alpha(x-\beta)} + 2\alpha\, e^{-\alpha(x-\beta)}\right\} = 0$ (équilibre)

$\Rightarrow e^{-2\alpha(x_0-\beta)} = e^{-\alpha(x_0-\beta)} \Rightarrow 2\alpha(x_0-\beta) = \alpha(x_0-\beta)$

$\Rightarrow \alpha(x_0-\beta) = 0 \Rightarrow \boxed{\beta = x_0}$

2) $\dfrac{dE_p}{dx} = 2\alpha D\, e^{-\alpha(x-\beta)}\left\{1 - e^{-\alpha(x-\beta)}\right\}$

$x < x_0 = \beta \Rightarrow x - \beta < 0 \Rightarrow e^{-\alpha(x-\beta)} > 1 \Rightarrow \dfrac{dV}{dx} < 0$

$x > x_0 \Rightarrow \dfrac{dV}{dx} > 0$

équilibre stable

3) a) $E_p(x) \simeq E_p(x_0) + \dfrac{x-x_0}{1!}E_p'(x_0) + \dfrac{(x-x_0)^2}{2!}E_p''(x_0) = -D + \varepsilon \times 0 + \dfrac{\varepsilon^2}{2}V''(x_0)$

$\dfrac{d^2E_p}{dx^2} = 2\alpha D\left\{2\alpha\, e^{-2\alpha(x-x_0)} - \alpha\, e^{-\alpha(x-x_0)}\right\}$

$\left(\dfrac{d^2E_p}{dx^2}\right)_{x_0} = 2\alpha^2 D \Rightarrow \boxed{E_p(x) \simeq -D + \alpha^2 D \varepsilon^2}$

b) $E = E_c + E_p$

$E_c = \frac{1}{2}m\dot{x}^2 = \frac{1}{2}m\dot{\varepsilon}^2 \Rightarrow \boxed{E \simeq \frac{1}{2}m\dot{\varepsilon}^2 - D + \alpha^2 D\varepsilon^2}$ à une cte près

Si l'énergie est constante $\dfrac{dE}{dt} = 0 \simeq m\ddot{\varepsilon}\dot{\varepsilon} + 2\alpha^2 D\dot{\varepsilon}\varepsilon$

$\Rightarrow \boxed{\ddot{\varepsilon} + \dfrac{2\alpha^2 D}{m}\varepsilon = 0}$

c) $\omega^2 = \dfrac{2\alpha^2 D}{m}$ $\quad T = \dfrac{2\pi}{\omega} \Rightarrow \boxed{T = \dfrac{2\pi}{\alpha}\sqrt{\dfrac{m_r}{2D}}}$

4) $D = -E_p(x_0) = 8.10^{-18}\,J$

$\alpha = \dfrac{2\pi}{T}\sqrt{\dfrac{m_r}{2D}} = 1{,}28.10^{10}\,m^{-1}$

Ionisation $\Rightarrow$ x passe de x_0 à ∞ $\Rightarrow E_{ion} = E_p(\infty) - E_p(x_0)$

$\Rightarrow E_{ion} = -E_p(x_0) = D = 8.10^{-18}\,J = 50\,eV$

5) a) $E(x_0) = 0 \Rightarrow E = \frac{1}{2}m\dot{\varepsilon}^2 + \alpha^2 D\varepsilon^2 = \frac{1}{2}m a^2\omega^2\cos^2(\omega t) + \alpha^2 D a^2\sin^2(\omega t)$

avec $\omega^2 = \dfrac{2\alpha^2 D}{m} \Rightarrow \frac{1}{2}m\omega^2 = \alpha^2 D$

$$\boxed{\mathcal{E} = \alpha^2 D a^2 = \frac{1}{2} m_2 \omega^2 a^2}$$

b) α) $\int_0^T p\,dx = \int_0^T m(\dot{x})^2 dt = \int_0^T m a^2 \omega^2 \cos^2(\omega t)\,dt = m a^2 \omega^2 \int_0^T \frac{1 + \cos(2\omega t)}{2}\,dt$

$= m a^2 \omega^2 \frac{T}{2} = m a^2 \omega^2 \frac{2\pi}{2\omega}$

$$= \boxed{\int_0^T p\,dx = \pi m_2 a^2 \omega}$$

β) $\mathcal{E} = \frac{1}{2} m_2 \omega^2 a^2 = \frac{1}{2} \omega \frac{1}{\pi} (n + 1/2) h$

$\Rightarrow \boxed{\mathcal{E} = \frac{\omega h}{2\pi} (n + 1/2)}$ discontinue

γ) Pour $n = 0$ $\quad \mathcal{E}_0 = \frac{\omega h}{4\pi} = \frac{h}{2T} \Rightarrow \underline{\mathcal{E}_0 = 6{,}62 \cdot 10^{-20}\ J = 0{,}41\ eV}$