Cinématique

1) $x = \rho \cos\varphi$

$y = \rho \sin\varphi$ $\Rightarrow$ $\boxed{\begin{array}{l} \rho = b e^{-kt} \\ \varphi = kt \end{array}}$

2) $\boxed{\rho = b e^{-\varphi}}$

3) Spirale dont les coordonnées du centre sont $(0,0,0)$

4) $\vec{OM} = \rho \vec{e_r}$

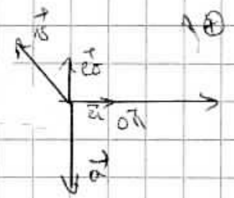
$\vec{v} = \frac{d\rho}{dt}\vec{e_r} + \rho\dot{\theta}\vec{e_\theta} = -kb e^{-kt}\vec{e_r} + kb e^{-kt}\vec{e_\theta}$

$$\boxed{\vec{v} = -k\rho\vec{e_r} + k\rho\vec{e_\theta}}$$

5) $\vec{a} = \cancel{k^2 b e^{-kt}\vec{e_r}} - k^2 b e^{-kt}\vec{e_\theta} - k^2 b e^{-kt}\vec{e_\theta} - \cancel{k^2 b e^{-kt}\vec{e_r}}$

$\vec{a} = -2k^2 b e^{-kt}\vec{e_\theta} = \boxed{-2k^2\rho\vec{e_\theta} = \vec{a}}$

6) $\vec{v} = k\rho(\vec{e_\theta} - \vec{e_r})$



$\alpha = \frac{\pi}{2} + \frac{\pi}{4}$ $\Rightarrow$ $\boxed{\alpha = \frac{3\pi}{4}}$

7) $\boxed{\beta = \frac{3\pi}{2}}$ ou $-\frac{\pi}{2}$

8) $\boxed{\gamma = \frac{3\pi}{4}}$ ou $-\frac{5\pi}{4}$

Régime sinusoïdal forcé en électricité

(2)

1°) $e = E\cos(\omega t + \varphi)$ avec $\omega = 2\pi f = 314,1\ \text{rad}\,s^{-1}$

$e(t=0) > 0 \Rightarrow E\cos\varphi > 0$

$\left(\frac{de}{dt}\right)_0 < 0 \Rightarrow -E\omega\sin\varphi < 0$

$\Rightarrow \boxed{e = 20\cos\left(314,1\,t + \pi/4\right)}$ convient

2°) Nécessairement pour les lois de Kirchhoff $\omega_1 = \omega_2 = \omega = 314,1\ \text{rad}\,s^{-1}$

3°) $\boxed{\begin{aligned} \underline{Z}_1 &= R + jL\omega \\ \underline{Z}_2 &= R + jL\omega + \frac{1}{jC\omega} \end{aligned}}$

4°) $\underline{Z} = \frac{\underline{Z}_1\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} = \frac{(R+jL\omega)\left(R+jL\omega+\frac{1}{jC\omega}\right)}{2R + 2jL\omega + \frac{1}{jC\omega}} = \boxed{\frac{(R+jL\omega)(1 - LC\omega^2 + jRC\omega)}{1 - 2LC\omega^2 + 2jRC\omega}}$

5°) $\underline{e} = (R + jL\omega)\,\underline{i}_1$

$\Rightarrow \underline{E} = (R + jL\omega)\,\underline{I}_1 \Rightarrow \boxed{\underline{I}_1 = \frac{\underline{E}}{R + jL\omega}}$

avec $\underline{E} = E\,e^{j\pi/4}$

et $\underline{E} = \left(R + jL\omega + \frac{1}{jC\omega}\right)\underline{I}_2 \Rightarrow \boxed{\underline{I}_2 = \frac{\underline{E}}{R + jL\omega + \frac{1}{jC\omega}}}$

6°) $\arg(\underline{e}) = \arg(R + jL\omega) + \arg(\underline{i}_1)$

$\omega t + \pi/4 = \arg(R + jL\omega) + \omega t + \varphi_{01} \Rightarrow \varphi_{01} = \pi/4 - \arg(R + jL\omega)$ avec $\varphi_1 = \varphi_{01} - \pi/4$

$\Rightarrow \boxed{\varphi_1 = - \text{Arctan}\left(\frac{L\omega}{R}\right)}$

$\arg(\underline{e}) = \arg\left(R + jL\omega + \frac{1}{jC\omega}\right) + \arg(\underline{i}_2)$

$\omega t + \pi/4 = \arg\left(R + jL\omega + \frac{1}{jC\omega}\right) + \omega t + \varphi_{02} \Rightarrow \varphi_{02} = \pi/4 - \arg\left(R + jL\omega + \frac{1}{jC\omega}\right)$ avec $\varphi_2 = \varphi_{02} - \pi/4$

$\Rightarrow \varphi_2 = - \text{Arctan}\left(\frac{L\omega - \frac{1}{C\omega}}{R}\right)$

$\Rightarrow \boxed{\varphi_2 = - \text{Arctan}\left(\frac{LC\omega^2 - 1}{RC\omega}\right)}$

7°) $\text{Arctan}\left(\frac{L\omega}{R}\right) = -\text{Arctan}\left(\frac{LC\omega^2 - 1}{RC\omega}\right)$

$\Rightarrow \frac{L\omega}{R} = \frac{1 - LC\omega^2}{RC\omega} \Rightarrow \boxed{2LC\omega^2 = 1}$

8°) $\left|\mathrm{Arg}(\underline{i_2}) - \mathrm{Arg}(\underline{i_1})\right| = \pi/2$

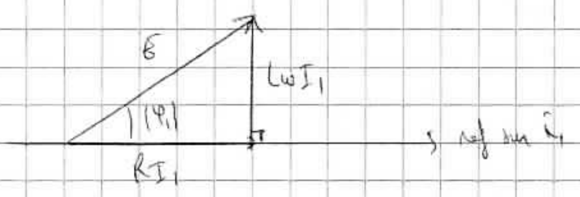
$$\Rightarrow \left| \mathrm{Arctan}\left(\frac{L\omega}{R}\right) - \mathrm{Arctan}\left(\frac{LC\omega^2-1}{RC\omega}\right)\right| = \pi/2$$

$$\Rightarrow \mathrm{Arctan}\left(\frac{L\omega}{R}\right) = \pi/2 + \mathrm{Arctan}\left(\frac{LC\omega^2-1}{RC\omega}\right) \quad \text{ou} \quad \mathrm{Arctan}\left(\frac{LC\omega^2-1}{RC\omega}\right) = \pi/2 + \mathrm{Arctan}\left(\frac{L\omega}{R}\right)$$

$$\Rightarrow \frac{LC\omega^2-1}{RC\omega} = -\frac{R}{L\omega}$$

$$\Rightarrow \boxed{R^2C = L(1-LC\omega^2)}$$

9°) $|\varphi_2 - \varphi_1| = \pi/2$ et $\varphi_2 = -\varphi_1$ $\Rightarrow$ $\varphi_2 = -\varphi_1 = \pm\frac{\pi}{4}$



$\varphi_1 = -\frac{\pi}{4}$

et $\varphi_2 = +\frac{\pi}{4}$

$$\Rightarrow \begin{cases} \varphi_{01} = \varphi_1 + \frac{\pi}{4} = 0 \\ \varphi_{02} = \varphi_2 + \frac{\pi}{4} = \frac{\pi}{2} \end{cases} \Rightarrow \boxed{\begin{cases} i_1 = I_{m1}\cos(\omega t) \\ i_2 = I_{m2}\cos(\omega t + \pi/2) = -I_{m2}\sin(\omega t) \end{cases}}$$

Étude de la résonance en amplitude du mvt

1°) $\underline{X}$ est l'amplitude complexe. Son module représente l'amplitude de l'oscillation, l'argument représente le déphasage des oscillations par rapport à la référence.

2°) $-\omega^2 \underline{x} + \frac{\omega_0}{Q} j\omega \underline{x} + \omega_0^2 \underline{x} = A_0 \exp(j\omega t) \exp(j\phi)$

$\underline{X} \cancel{\exp(j\omega t)} \left(\omega_0^2 - \omega^2 + j\frac{\omega\omega_0}{Q}\right) = A_0 \cancel{\exp(j\omega t)} \exp(j\phi)$

$\Rightarrow \boxed{|\underline{X}| = \frac{A_0}{\sqrt{(\omega_0^2-\omega^2)^2 + \left(\frac{\omega\omega_0}{Q}\right)^2}}}$ (avec $A_0 > 0$)

3°) $\omega \to 0 \quad |\underline{X}| \to \frac{A_0}{\omega_0^2}$

graphe à rajouter

$\omega \to \infty \quad |\underline{X}| \to 0$

4°) $f(\omega) = (\omega_0^2 - \omega^2)^2 + \frac{\omega^2\omega_0^2}{Q^2}$

$\frac{df}{d\omega} = 2(-2\omega)(\omega_0^2-\omega^2) + 2\frac{\omega_0^2\omega}{Q^2} = 2\omega\left[-2\omega_0^2 + 2\omega^2 + \frac{\omega_0^2}{Q^2}\right] = 0$

$\Rightarrow 2\omega^2 = 2\omega_0^2 - \frac{\omega_0^2}{Q^2} = \omega_0^2\left(2 - \frac{1}{Q^2}\right)$

$\Rightarrow \omega^2 = \omega_0^2\left(1 - \frac{1}{2Q^2}\right)$

$\omega^2 > 0 \Rightarrow 1 > \frac{1}{2Q^2} \Rightarrow \boxed{Q > \frac{1}{\sqrt{2}}}$

5°) $\boxed{\omega_R = \omega_0\sqrt{1 - \frac{1}{2Q^2}}}$

6°) $\omega_0^2 - \omega_R^2 = \frac{\omega_0^2}{2Q^2}$

$X_1 = \frac{A_0}{\sqrt{\left(\frac{\omega_0^2}{2Q^2}\right)^2 + \frac{\omega_0^4}{Q^2}\left(1-\frac{1}{2Q^2}\right)}} = \frac{A_0}{\sqrt{\frac{\omega_0^4}{Q^2}\left(1 - \frac{1}{4Q^2}\right)}} \Rightarrow \boxed{X_1 = \frac{QA_0}{\omega_0^2\sqrt{1-\frac{1}{4Q^2}}}}$

7°) $X_c = \frac{X_1}{\sqrt{2}} \Rightarrow \frac{A_0}{\sqrt{(\omega_0^2-\omega^2)^2 + \left(\frac{\omega\omega_0}{Q}\right)^2}} = \frac{QA_0}{\omega_0^2\sqrt{2}\sqrt{1-\frac{1}{4Q^2}}}$

$\Rightarrow (\omega_0^2-\omega^2)^2 + \frac{\omega_0^2\omega^2}{Q^2} = \frac{2\omega_0^4}{Q^2}\left(1 - \frac{1}{4Q^2}\right)$

$\Rightarrow \omega^4 - 2\omega^2\omega_0^2 + \omega_0^4 + \frac{\omega_0^2\omega^2}{Q^2} = \frac{2\omega_0^4}{Q^2}\left(1 - \frac{1}{4Q^2}\right)$

$\Rightarrow \omega^4 + \omega^2\left(\frac{\omega_0^2}{Q^2} - 2\omega_0^2\right) + \omega_0^4 - \frac{2\omega_0^4}{Q^2}\left(1 - \frac{1}{4Q^2}\right) = 0$

$\Rightarrow \boxed{\omega^4 + \omega^2\omega_0^2\left(\frac{1}{Q^2} - 2\right) + \omega_0^4\left(1 - \frac{2}{Q^2} + \frac{1}{2Q^4}\right) = 0}$

8°) $\Delta = \omega_0^4\left(\frac{1}{Q^2}-2\right)^2 - 4\omega_0^4\left(1-\frac{2}{Q^2}+\frac{1}{2Q^4}\right)$

$$= \omega_0^4\left[\frac{1}{Q^4}-\frac{4}{Q^2}+4-4+\frac{8}{Q^2}-\frac{2}{Q^4}\right] = \omega_0^4\left[\frac{4}{Q^2}-\frac{1}{Q^4}\right] = \frac{\omega_0^4}{Q^2}\left[4-\frac{1}{Q^2}\right]$$

$$\Rightarrow \boxed{\omega_i^2 = \omega_0^2\left(1-\frac{1}{2Q^2}\right) \pm \frac{\omega_0^2}{Q}\sqrt{1-\frac{1}{4Q^2}}} \qquad \left(\exists \text{ si } Q > \sqrt{1+\frac{1}{Q^2}}\right)$$

9°) Si Q est intermédiaire $\quad \omega_i^2 \simeq \omega_0^2\left(1 \pm \frac{1}{Q}\right)$

$$\rightarrow \boxed{\omega_i = \omega_0\sqrt{1 \pm \frac{1}{Q}}} \simeq \omega_0\left(1 \pm \frac{1}{2Q}\right)$$

10°) $\Delta\omega = \omega_2 - \omega_1 = \frac{1}{Q}\omega_0 \quad \Rightarrow \quad \boxed{\Delta\omega = \frac{\omega_0}{Q}}$

11°) $\boxed{f_r \simeq 539{,}6 \text{ Hz}}$

fréquence de coupure $\quad f_1 \simeq 539{,}5 \text{ Hz}$

$f_2 \simeq 539{,}7 \text{ Hz} \qquad \Delta f = 0{,}2 \text{ Hz} \Rightarrow \Delta\omega = 1{,}26 \text{ rad.s}^{-1}$

$\omega_r = \omega_0\sqrt{1-\frac{1}{2Q^2}} \simeq \omega_0 \quad \Rightarrow \omega_0 \simeq 2\pi f_r = 3390{,}4 \text{ rad.s}^{-1}$

$$Q = \frac{\omega_0}{\Delta\omega} = \underline{2690}$$

12°) $0{,}1 < f_r < 2{,}1$ Hz

Mais $0{,}5\% \times 539{,}6 = 2{,}7$ Hz $\quad$ <u>imprécision trop forte</u> pour tracer le graphe donné