Homogénéité

1) $d = \sqrt{\frac{2D}{\omega}} \Rightarrow D = \frac{\omega}{2} d^2 \Rightarrow [D] = L^2 T^{-1}$

$\mu = \frac{m}{V} \Rightarrow [\mu] = M L^{-3}$

$r = k D^\alpha \mu^\beta m^\gamma \Rightarrow T = (L^2 T^{-1})^\alpha (M L^{-3})^\beta (M)^\gamma$

$$\Rightarrow \begin{cases} 0 = 2\alpha - 3\beta & \text{en } L \\ 1 = -\alpha & \text{en } T \\ 0 = \beta + \gamma & \text{en } M \end{cases} \Rightarrow \begin{cases} \alpha = -1 \\ \beta = \frac{2}{3}\alpha = -\frac{2}{3} \\ \gamma = -\beta = \frac{2}{3} \end{cases}$$

$$\boxed{r = k D^{-1} \mu^{-2/3} m^{2/3}} \qquad r = k \frac{m^{2/3}}{D \mu^{2/3}} = \frac{k}{D} \sqrt[3]{\left(\frac{m}{\mu}\right)^2}$$

2) $\frac{r_1}{r_2} = \frac{D_1^{-1} \mu_1^{-2/3} m_1^{2/3}}{D_2^{-1} \mu_2^{-2/3} m_2^{2/3}} = \left(\frac{m_1}{m_2}\right)^{2/3} \Rightarrow \boxed{r_1 = \left(\frac{m_1}{m_2}\right)^{2/3} r_2}$

$r_1 = 112{,}6 \text{ min} = \underline{1^h\,52'}$

3) $\chi = -\frac{1}{V}\frac{dV}{dP} \Rightarrow [\chi] = [P]^{-1} = \left[\frac{S}{F}\right] = \frac{L^2}{M L T^{-2}} = M^{-1} L T^{+2}$

$[c] = L T^{-2}$

$c = \chi^\alpha \mu^\beta \Rightarrow L T^{-1} = (M^{-1} L T^{+2})^\alpha (M L^{-3})^\beta$

$$\Rightarrow \begin{cases} 1 = \alpha - 3\beta \\ 0 = -\alpha + \beta \\ -1 = +2\alpha \end{cases} \Rightarrow \begin{cases} \alpha = -\frac{1}{2} \\ \beta = -\frac{1}{2} \end{cases} \Rightarrow \boxed{c = \frac{1}{\sqrt{\chi \mu}}}$$

4) $\underline{c = 342{,}6 \text{ m.s}^{-1}}$

5) $\eta = \frac{f}{6\pi\eta r v} \Rightarrow [\eta] = \frac{M L T^{-2}}{L (L T^{-1})} = M L^{-1} T^{-1}$

$v = \rho g^\alpha r^\beta \eta^\gamma \Rightarrow L T^{-1} = (M L^{-3}) (L T^{-2})^\alpha (L)^\beta (M L^{-1} T^{-1})^\gamma$

$$\Rightarrow \begin{cases} 1 = -3 + \alpha + \beta - \gamma & \text{en } L \\ 0 = 1 + \gamma & \text{en } M \\ -1 = -2\alpha - \gamma & \text{en } T \end{cases} \rightarrow \begin{cases} \gamma = -1 \\ \alpha = 1 \\ \beta = 2 \end{cases} \qquad \boxed{v = \frac{\rho g r^2}{\eta}}$$

6) $\underline{v = 0{,}1 \text{ m.s}^{-1}}$

7) $f = G \frac{m m'}{r^2} \Rightarrow [G] = \frac{(MLT^{-2}) L^2}{M^2} = M^{-1} L^3 T^{-2}$

$T = k\, G^{\alpha} R^{\beta} \rho^{\gamma}$

$\Rightarrow \quad T = (M^{-1} L^3 T^{-2})^{\alpha} (L)^{\beta} (ML^{-3})^{\gamma}$

$$\Rightarrow \begin{cases} 1 = -2\alpha & \text{en } T \\ 0 = -\alpha + \gamma & \text{en } M \\ 0 = 3\alpha + \beta - 3\gamma & \text{en } L \end{cases} \Rightarrow \begin{cases} \alpha = -1/2 \\ \gamma = -1/2 \\ \beta = 0 \end{cases}$$

$$\Rightarrow \boxed{T = \frac{k}{\sqrt{G\rho}}}$$

8) $k = T\sqrt{G\rho}$ avec $T = 4,6 . 10^8$ s

$m = 5 \times M_{soleil} = 9,945 . 10^{30}$ kg

$V = 4/3 \, \pi R^3 = 1,04 . 10^{32}$ m^3

$\Rightarrow \rho = 0,0952$ kg m^{-3}

$\Rightarrow \underline{k = 1168}$

Oscillateur harmonique

1) Système : masse m $\qquad \vec{OM} = x\vec{i}$

Référentiel terrestre supposé galiléen $\qquad \vec{v} = \dot{x}\vec{i}$

Forces appliquées $\vec{P} = -mg\vec{j} \qquad \vec{T} = -k(l-l_0)\vec{i} \qquad \vec{a} = \ddot{x}\vec{i}$

$\vec{R} = R\vec{j}$

PFD $-mg\vec{j} + R\vec{j} - kx\vec{i} = m\ddot{x}\vec{i} \Rightarrow -kx = m\ddot{x}$

$$\Rightarrow \boxed{\ddot{x} + \frac{k}{m}x = 0}$$

2) $\boxed{E = E_c + E_p = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} k x^2}$

3) $E = \text{cste}$ (car forces conservatives ou ⊥ déplacement)

$\frac{dE}{dt} = 0 = m\dot{x}\ddot{x} + k\dot{x}x$

$\dot{x} \neq 0 \Rightarrow \boxed{m\ddot{x} + kx = 0}$

4) $x = A\cos(\omega t) + B\sin(\omega t)$

$$\left.\begin{array}{l} \text{à } t=0 \quad x_0 = A \\ v_0 = (\dot{x})_0 = B\omega \end{array}\right\} \Rightarrow \boxed{x = x_0\cos(\omega t) + \frac{v_0}{\omega}\sin(\omega t)}$$

5) $E = \frac{1}{2} m\left(-\omega x_0\sin(\omega t) + v_0\cos(\omega t)\right)^2 + \frac{1}{2} k\left(x_0\cos(\omega t) + \frac{v_0}{\omega}\sin(\omega t)\right)^2$ avec $\omega = \sqrt{\frac{k}{m}}$

$= \frac{1}{2} k x_0^2\sin^2(\omega t) - m\omega \cancel{x_0 v_0\cos(\omega t)}\sin(\omega t) + \frac{1}{2} m v_0^2\cos^2(\omega t) + \frac{1}{2} k x_0^2\cos^2(\omega t) + k\cancel{\frac{x_0 v_0}{\omega}\cos(\omega t)}\sin(\omega t)$

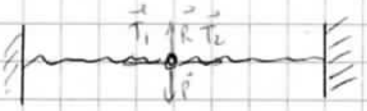
$+ \frac{1}{2} m v_0^2\sin^2(\omega t)$

$= \boxed{\frac{1}{2} k x_0^2 + \frac{1}{2} m v_0^2 = E}$

Comment mesurer la masse d'un astronaute ?

1) Système = cabine

Référentiel terrestre supposé galiléen



$\vec{OM} = x\vec i$

$\vec v = \dot x \vec i$

$\vec a = \ddot x \vec i$

Forces appliquées : Poids $\vec P = -(M+m)g\,\vec j$

Réaction $\vec R = R\vec j$

Tensions $\vec T_1 = -k(l-l_0)\vec i = -k(l_{eq}+x-l_0)\vec i$

$\vec T_2 = -k(l-l_0)(-\vec i) = +k(l_{eq}-x-l_0)\vec i$

PFD $(M+m)\ddot x\,\vec i = -(M+m)g\,\vec j + R\vec j - k(l_{eq}+x-l_0)\vec i + k(l_{eq}-x-l_0)\vec i$

$\Rightarrow (M+m)\ddot x = -2kx$

$\Rightarrow \boxed{\ddot x + \frac{2k}{M+m}x = 0}$

2) $\omega_0 = \sqrt{\frac{2k}{M+m}}$ $\quad x = A\cos(\omega_0 t) + B\sin(\omega_0 t)$

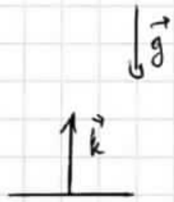
$$\left.\begin{array}{l} \text{à } t=0 \quad x = X_m = A \\ \dot x_0 = 0 = B\omega \end{array}\right\} \Rightarrow \boxed{x = X_m\cos(\omega_0 t)}$$

3) $T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{M+m}{2k}} \rightarrow M+m = 2k\frac{T^2}{4\pi^2} \rightarrow \boxed{M = \frac{kT^2}{2\pi^2} - m}$

AN $\underline{M = 81,3 \text{ kg}}$

4) oui g n'intervient pas dans l'expression de M

Système de 2 masses liées par un ressort vertical



1°) Système = masse m

$$\vec{P} + \vec{T} = \vec{0} \Rightarrow -mg\vec{k} - k(z_e - l_0)\vec{k} = \vec{0}$$

$$\Rightarrow -mg - k(z_e - l_0) = 0 \Rightarrow \boxed{z_e = l_0 - \frac{mg}{k}}$$

2°) $\vec{P} + \vec{T} = m\vec{a} \Rightarrow -mg - k(z - l_0) = m\ddot{z}$

$$\Rightarrow \boxed{\ddot{z} + \frac{k}{m} z = \frac{k}{m} l_0 - g = \frac{k}{m} z_e}$$

3°) $u = z - z_e \Rightarrow \ddot{u} = \ddot{z} \Rightarrow \boxed{\ddot{u} + \omega^2 u = 0}$

4°) a) $u = A\cos(\omega t) + B\sin(\omega t)$

à $t = 0$ $\left.\begin{array}{l} u(0) = z_0 - z_e = A \\ \dot{u}(0) = \dot{z}(0) = 0 = B\omega \end{array}\right\} \Rightarrow \boxed{u = (z_0 - z_e)\cos(\omega t)}$

b) $\boxed{z = z_e + (z_0 - z_e)\cos(\omega t)}$

c) Maximum pour $\cos(\omega t) = \pm 1 \Rightarrow z_m = 2z_e - z_0$ ou $z_m = z_0$

avec $z_e > z_0 \Rightarrow \boxed{z_m = 2z_e - z_0}$

5°) a) $\vec{T}_m = -k(z_m - l_0)\vec{k} = -k(2z_e - z_0 - l_0)\vec{k} = -k\left(2l_0 - \frac{2mg}{k} - z_0 - l_0\right)\vec{k}$

$$\Rightarrow \boxed{\vec{T}_m = -k\left(l_0 - z_0 - \frac{2mg}{k}\right)\vec{k}}$$

b)

Il faut $\vec{T}'_m + \vec{P}' > 0$ avec $\vec{T}'_m = -\vec{T}_m$ (action, réaction)

$$k\left(l_0 - z_0 - \frac{2mg}{k}\right)\vec{k} - m'g\vec{k} > 0$$

$$\Rightarrow k\left(l_0 - z_0 - \frac{2mg}{k}\right) > m'g$$

$$\Rightarrow \boxed{l_0 - \frac{2mg}{k} - \frac{m'g}{k} > z_0}$$

Équilibres chimiques

I) 1) $Q_i = \dfrac{[BO_2^-]_0 [NH_4^+]_0}{[HBO_2]_0 [NH_3]_0} = 0 < K$ sens 1 →

2) $HBO_2 + NH_3 = BO_2^- + NH_4^+$

EI	0,1	0,1	0	0
éq	$0{,}1 - x_{eq}$	$0{,}1 - x_{eq}$	x_{eq}	x_{eq}

$\Rightarrow \boxed{K = \dfrac{x_{eq}^2}{(0{,}1 - x_{eq})^2}}$

3) $x_{eq} = \sqrt{K}\,(0{,}1 - x_{eq}) \rightarrow \boxed{x_{eq} = \dfrac{0{,}1\sqrt{K}}{1 + \sqrt{K}}}$ AN $x_{eq} = 5{,}3 \cdot 10^{-2}$

$\Rightarrow [BO_2^-] = [NH_4^+] = \underline{5{,}3 \cdot 10^{-2}\ mol\ L^{-1}}$

$[HBO_2] = [NH_3] = \underline{4{,}7 \cdot 10^{-2}\ mol\ L^{-1}}$

4) équilibrée

5) $K = \dfrac{(0{,}1 + x_{eq})^2}{(0{,}1 - x_{eq})^2} \Rightarrow x_{eq} = \dfrac{0{,}1(\sqrt{K} - 1)}{1 + \sqrt{K}} = 6{,}5 \cdot 10^{-3}$

$[BO_2^-] = [NH_4^+] = \underline{0{,}106\ mol\ L^{-1}}$

$[HBO_2] = [NH_3] = \underline{9{,}3 \cdot 10^{-2}\ mol\ L^{-1}}$

équilibre encore

II) 1) $Q = \dfrac{a_{O_2}\, a_{Cu_2O}^2}{a_{CuO}^4} = a_{O_2}$ car solides

$Q = \dfrac{P_{O_2}}{P^\circ}$ avec $P^\circ = 1$ bar

$P_{O_2} = \dfrac{nRT}{V}$ (donne P en Pa et pas en bar!)

$$\boxed{\begin{array}{ccc} Q_a = 0{,}108 & \text{et} & Q_b = 0{,}216 \\ \xrightarrow{1} & & \xleftarrow{2} \end{array}}$$

2) $K = \dfrac{(n + \xi)RT}{V\ P^\circ} \Rightarrow \xi = \dfrac{KVP^\circ}{RT} - n$

avec $P^\circ = 1\ bar = 10^5\ Pa$

$V = 10\ L = 10^{-2}\ m^3$

$\Rightarrow \xi = 8{,}5 \cdot 10^{-3}\ mol$

$4CuO = 2Cu_2O + O_2$

EI	0,1	10^{-3}	n
éq	$0{,}1 - 4\xi$	$10^{-3} + 2\xi$	$n + \xi$

$$\Rightarrow \boxed{\begin{array}{l} n_{O_2} = 1{,}65 \cdot 10^{-2}\ mol \\ n_{Cu_2O} = 1{,}8 \cdot 10^{-2}\ mol \\ n_{CuO} = 6{,}6 \cdot 10^{-2}\ mol \end{array}}$$

3) $\xi = -1{,}5 \cdot 10^{-3}$ (sens 2)

$n_{O_2} = 1{,}85 . 10^{-2}$ mol

$n_{CuO} = -4{,}9 . 10^{-4}$ mol < 0 impossible, on ne dispose pas d'assez de CuO, la réaction s'arrête avant

$n_{Cu_2O} = 0{,}606$ mol

$$\Rightarrow \boxed{\begin{array}{l} n_{CuO} = 0 \qquad \text{et } \xi = -0{,}5 . 10^{-3} \\ n_{O_2} = 2 . 10^{-2} - 0{,}5 . 10^{-3} = 0{,}0195 \text{ mol} \\ n_{Cu_2O} = 0{,}602 \text{ mol} \end{array}}$$

III) 1) $K = \left(\frac{a_{FeCl_6}}{a_{FeCl_3}^2} \right)_{éq} = \boxed{\left(\frac{p_{Fe_2Cl_6} \, p^0}{p_{FeCl_3}^2} \right)_{éq} = K}$

2) $p_{FeCl_6} = \frac{n_{FeCl_6}}{n_{tot}} p$

$p_{FeCl_3} = \frac{n_{FeCl_3}}{n_{tot}} p$

$$\boxed{Q = \frac{n_{FeCl_6} \, n_{tot}}{n_{FeCl_3}^2} \frac{p^0}{p}}$$

$$Q_0 = \frac{n_1 \, 2n_1}{n_1^2} \frac{p^0}{2p^0} = \underline{1}$$

3) $Q_0 < K \quad \xrightarrow[1]{}$

4)

	$2FeCl_3$	$\rightleftharpoons FeCl_6$	$V_{tot\,gaz}$
EI	n	0	n
éq	$n - 2\xi$	ξ	$n - \xi$

$$K = \frac{\xi (n - \xi)}{(n - 2\xi)^2} \frac{p^0}{2p^0}$$

$$\Rightarrow \frac{\xi/n \, (1 - \xi/n)}{(1 - 2\xi/n)^2} \frac{1}{2} = K$$

Et pour $x = \xi/n$ $\quad \frac{x(1-x)}{2(1-2x)^2} = K$

$\Rightarrow x - x^2 = 2K(1 - 4x + 4x^2) = 2K - 8Kx + 8Kx^2$

$\Rightarrow (8K+1)x^2 - (8K+1)x + 2K = 0$

$\Rightarrow 167{,}4 x^2 - 167{,}4 x + 41{,}6 = 0$

$\underline{x = 0{,}46}$ ou $x = 0{,}53$

(premier arrivé)